Force on Brownian Objects in Non-Equilibrium Steady States

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Physica A (arXiv:1204.6536v1)

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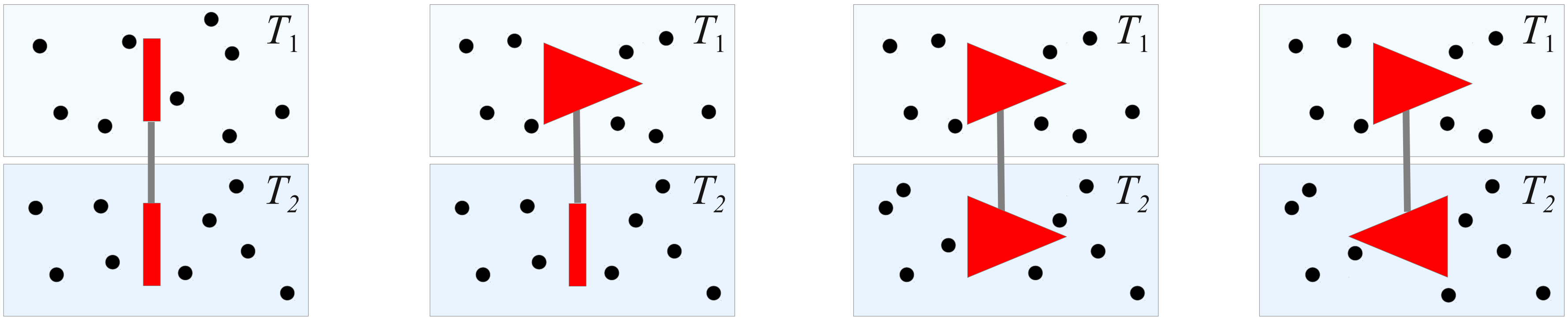
9-15 September, 2012 (Capri, Italy)

Brownian Motors



$\langle V \rangle = 0$

$\langle V \rangle < 0 \quad (T_1 > T_2)$
$\langle V \rangle > 0 \quad (T_1 < T_2)$

$\langle V \rangle > 0 \quad (T_1 \neq T_2)$

$\langle V \rangle < 0 \quad (T_1 > T_2)$
$\langle V \rangle > 0 \quad (T_1 < T_2)$

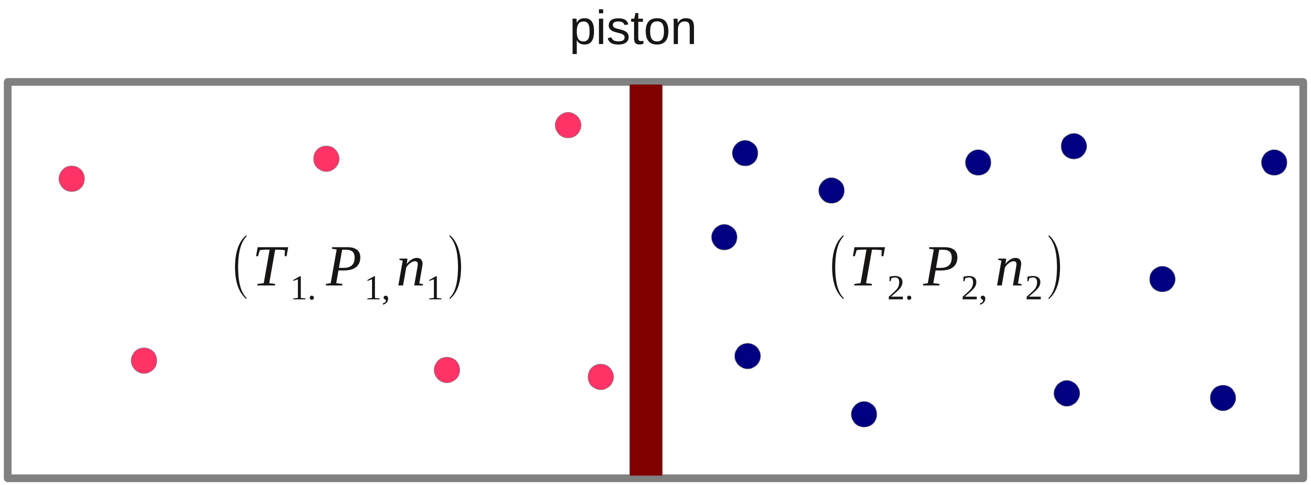
- Boltzmann-Master Equation (lengthy calculation)
- Molecular Dynamics Simulation

Intuitive Explanation?

Van den Broeck and Kawai, Phys. Rev. Lett. **93** (2004), 090601

Van den Broeck,Meurs, andKawai, New J. Phys. 7 (2005), 10

Adiabatic Piston



$$P_1 = P_2\,, \quad T_1 > T_2\,, \quad n_1 < n_2$$

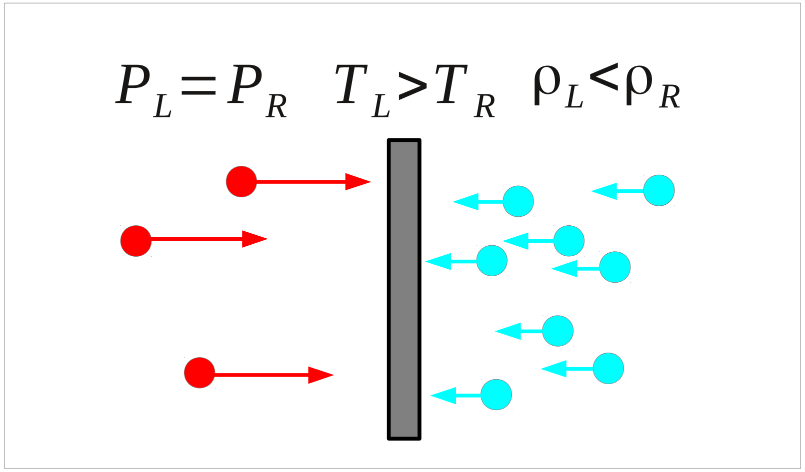
Does the piston move? → Yes, it does.

In which direction? → Toward the higher temperature.

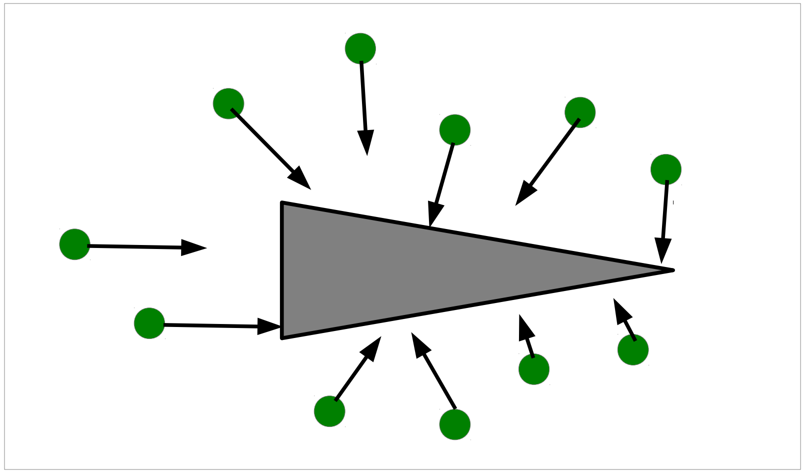
Intuitive Explanation?
Any Relation with the Brownian Motors?

Piasecki and Gruber, Physica A 265 (1999), 463, Gruber and Piasecki, Physica A 268 (1999), 412
Kestmont et al. , EPL 49 (2000), 143

What symmetry is Broken?



Large Fluctuation $\langle F_L \rangle = \langle F_R \rangle$ Small Fluctuation



- Temperature
- Density
- Geometric Shape
- **Fluctuation**

Utilizing the Fluctuation Asymmetry

Time Scale → $\frac{m}{M}$

Breaking Detailed Balance → Dissipation

Can Langevin theory explain these motions?

Lengevin Theory → $o\left(\frac{m}{M}\right)$

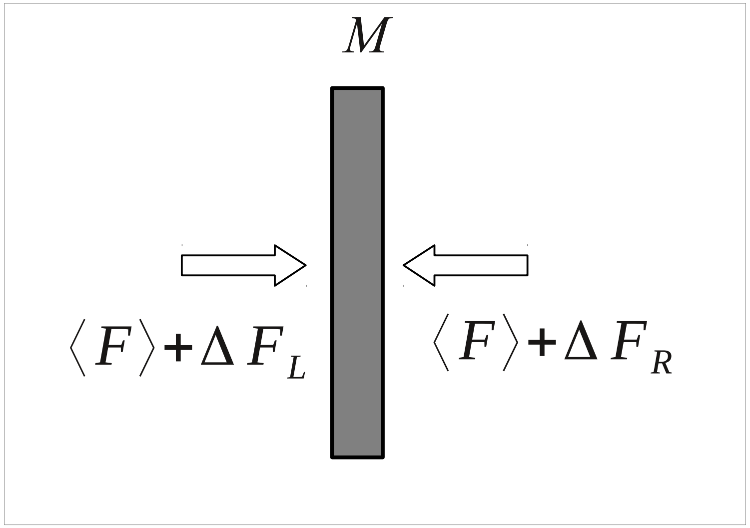
m=mass of gas particles
M=mass of Brownian objects

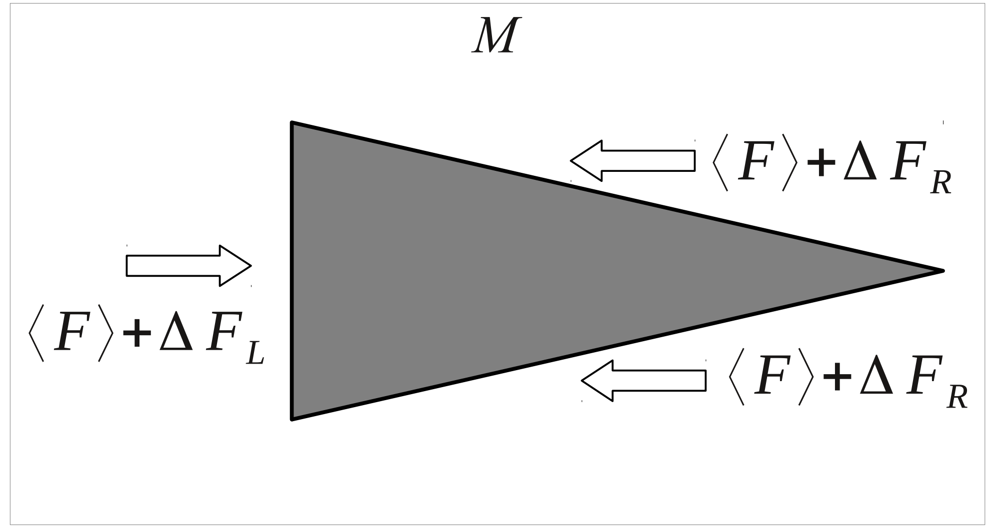
Brownian Motors
Adiabatic Piston → $o\left(\left[\frac{m}{M}\right]^2\right)$

Non-Linear Langevin Theory → As complicated as other methods and not very intuitive.

For adiabatic piston,
Plyukhin and Schofield, Phys. Rev. E **69** (2004), 021112

What is Missing in Linear Langevin Theory





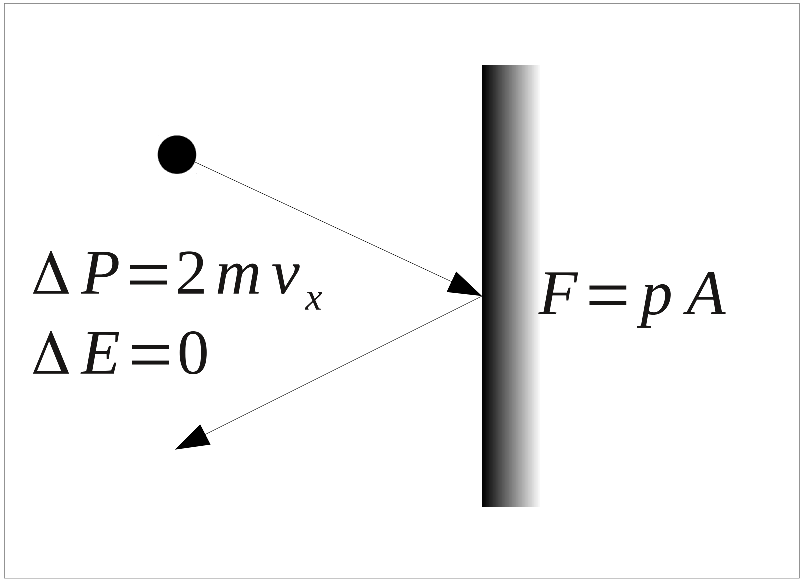
$$M\frac{\mathrm{d}V}{\mathrm{d}t}=-\gamma V+\sqrt{2\mathrm{D}}\,\xi(t) \qquad (D=\gamma k T)$$

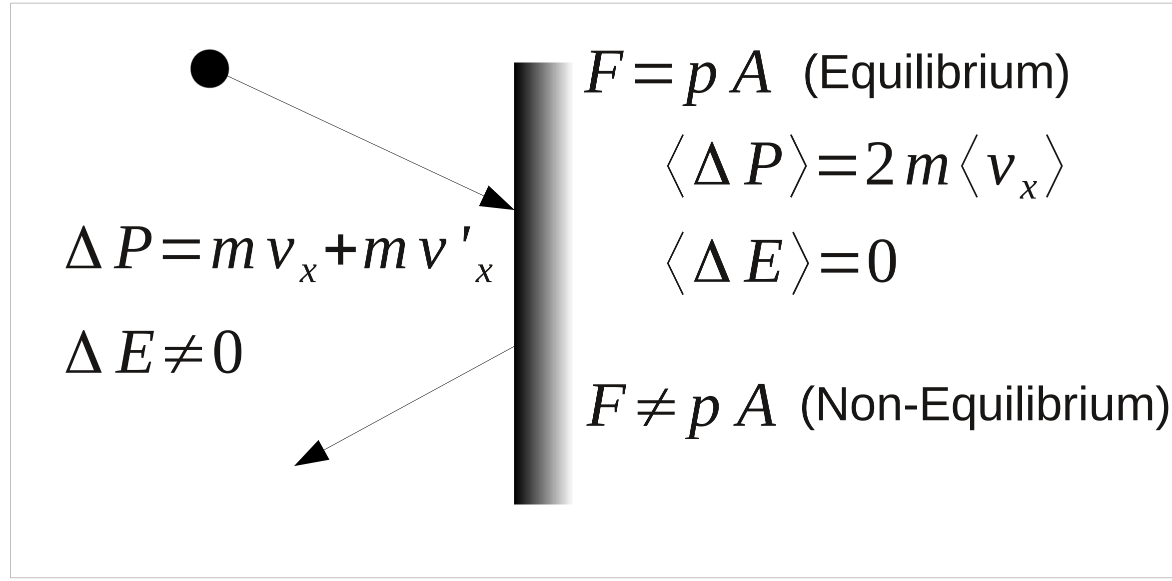
Energy: $\mathrm{d}\langle Q\rangle=\langle(-\gamma V+\sqrt{2\mathrm{D}}\,\xi(t))\circ \mathrm{d}X(t)\rangle$

Momentum: $\mathrm{d}\langle P\rangle=\langle(-\gamma V+\sqrt{2\mathrm{D}}\,\xi(t))\mathrm{d}t\rangle=-\gamma\langle V\rangle \mathrm{d}t$

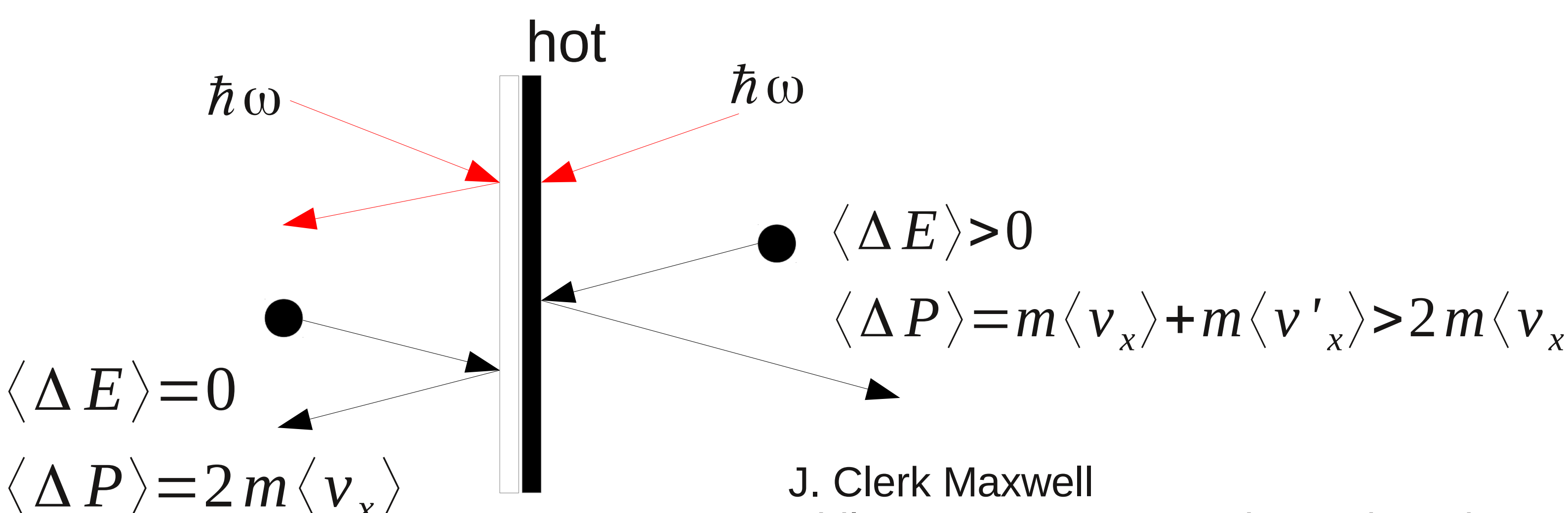
Asymmetry is not dictated in the linear Langevin theory

Pressure*Area ≠ Force on Wall



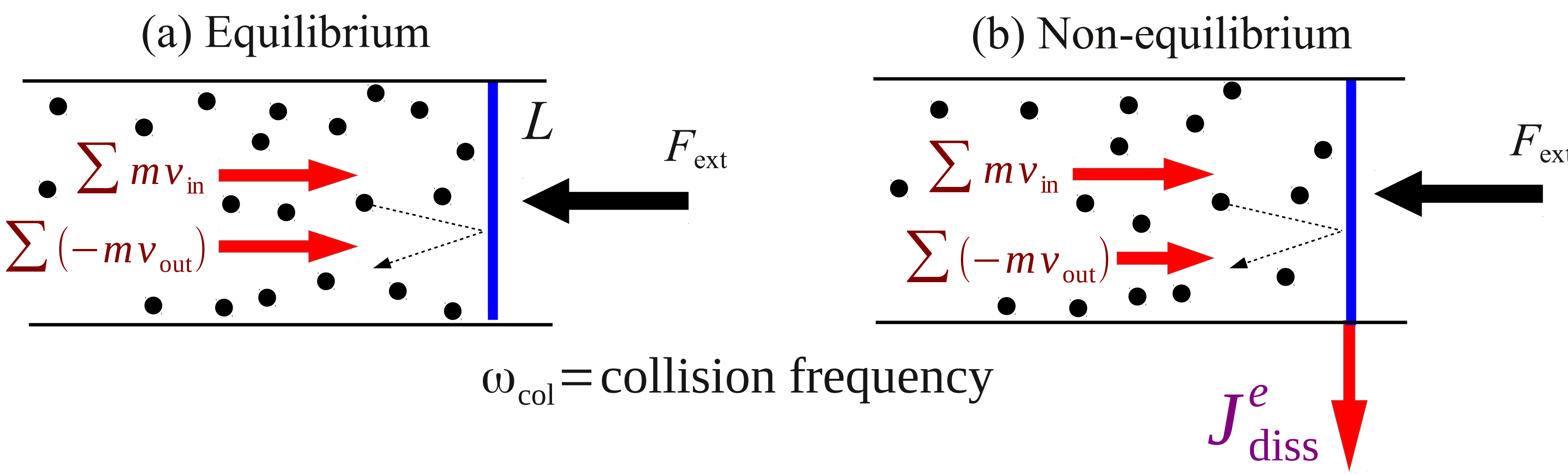


radiometer



J. Clerk Maxwell
Phil. Trans. R. Soc. Lond. **170** (1879), 231

Momentum Deficit due to Dissipation



ω_{col} = collision frequency

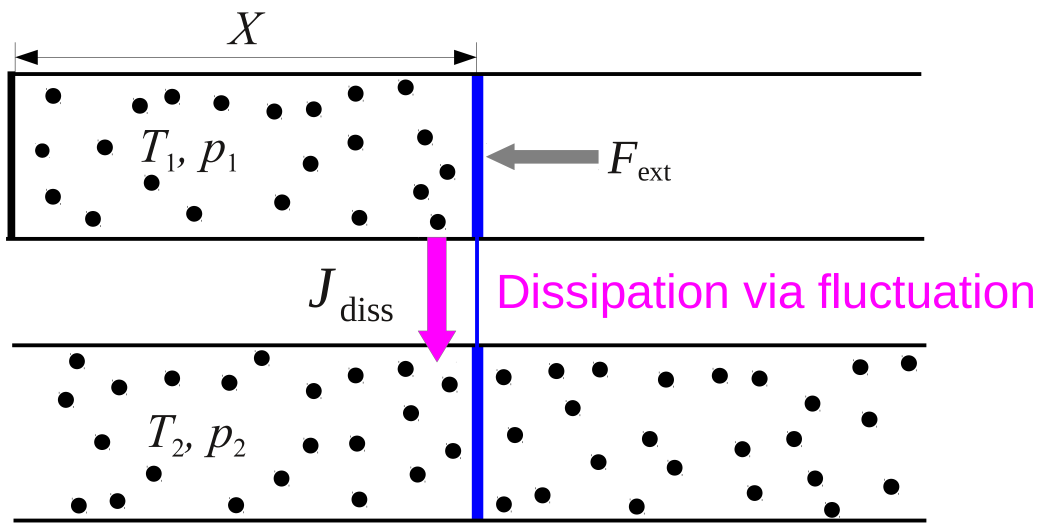
$$-F=(m v_{th}+m|v'|)\,\omega_{col}=(2 m v_{th}+m|v'|-m v_{th})\,\omega_{col}=p L+F_{MDD}$$

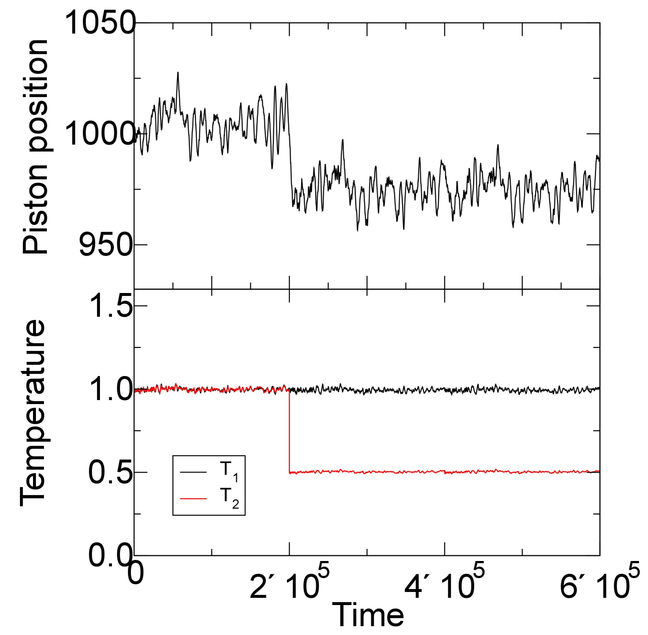
$$F_{MDD}=(m|v'|-m v_{th})\,\omega_{col} \qquad p L=2 m v_{th}\,\omega_{col}$$

$$\left(\frac{1}{2} m v_{th}^{2}-\frac{1}{2} m|v'|^{2}\right)\omega_{col}=J_{diss} \xrightarrow[v_{th}\sim|v|]{} (m v_{th}-m|v'|)\,\omega_{col}\approx\frac{J_{diss}}{v_{th}}$$

$$F_{MDD}\approx-\frac{J_{diss}}{v_{th}} \xrightarrow[\text{Hard disk gas}]{} F_{MDD}=-\sqrt{\frac{\pi}{8}}\frac{J_{diss}}{v_{th}}$$

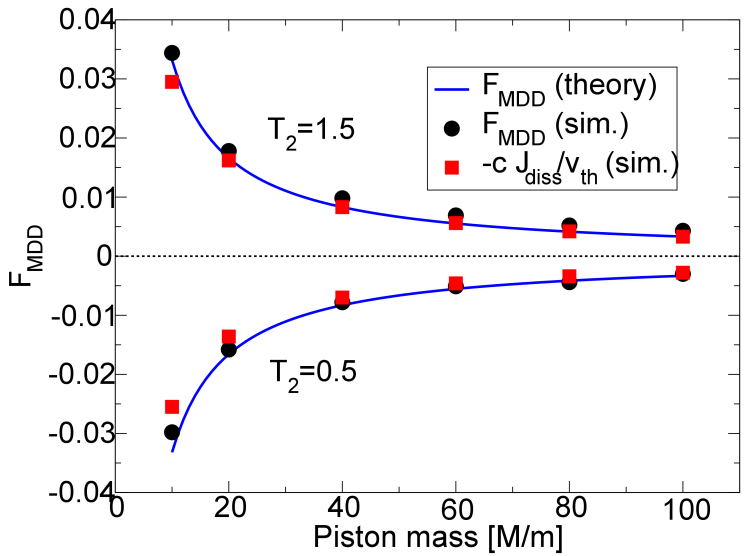
Simple Model 1: Shared Brownian Piston





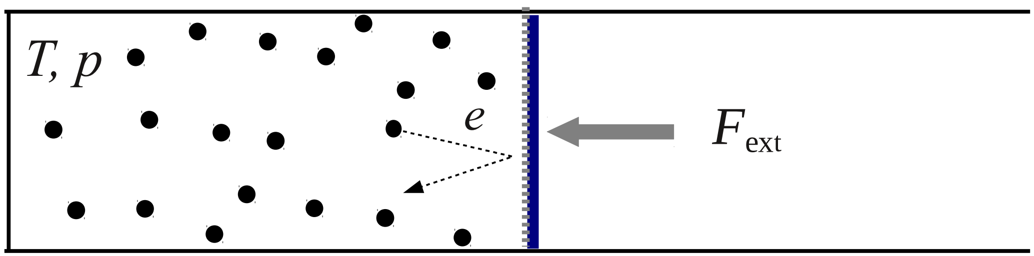
$$F_{\mathrm{MDD}} = -\sqrt{\frac{\pi}{8}}\frac{J_{\mathrm{diss}}}{v_{\mathrm{th}}}$$

Langevin Theory: $$J_{\mathrm{diss}} = \sqrt{\frac{\pi}{8}}\frac{kT_1 - kT_2}{M\left(\gamma_1^{-1}+\gamma_2^{-1}\right)}$$



$$\frac{F_{\mathrm{MDD}}}{L} = -\frac{2\rho_1\rho_2}{\rho_1+2\rho_2}\frac{m}{M}\left(kT_1 - kT_2\right)$$

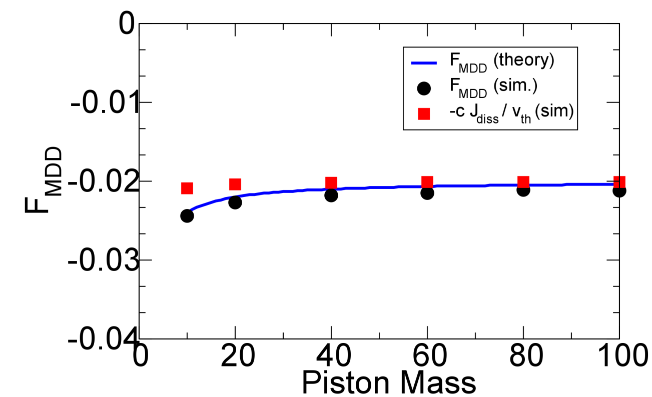
Simple Model 2: Inelastic Brownian Piston



e=restitution coefficient

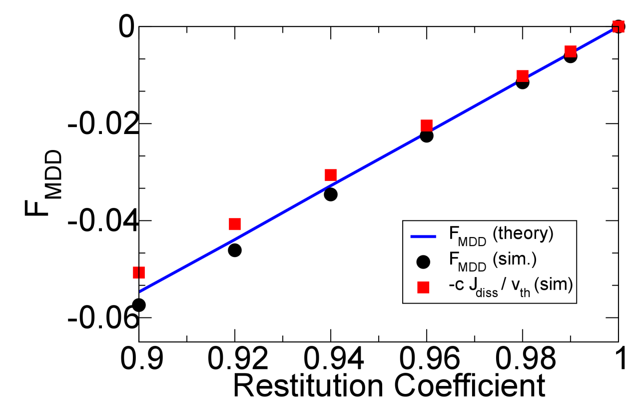
House keeping dissipation

$$J_{\text{diss, hk}} = (1-e)\sqrt{\frac{2}{\pi}}\, v_{\text{th}}\, p\, L \longrightarrow \frac{F_{\text{MDD,hk}}}{L} = -\frac{1}{2}(1-e)\, p$$

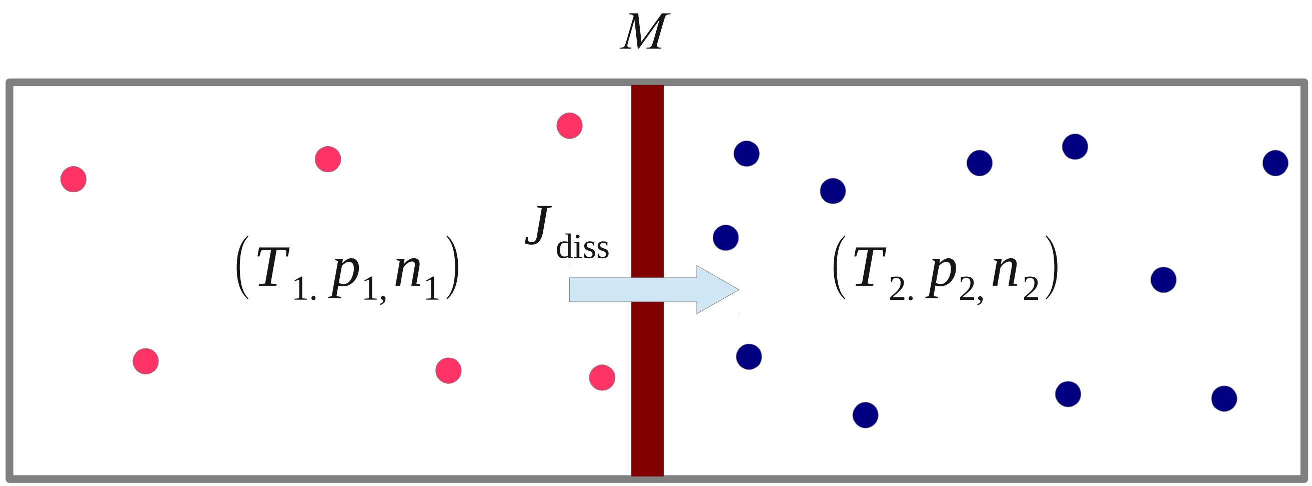


Excess dissipation

$$J_{\text{diss, ex}} = (1-e)\frac{\gamma}{M}\, v_{\text{th}}\, p\, L \longrightarrow \frac{F_{\text{MDD,hk}}}{L} = -\frac{m}{M}(1-e)\, p$$



Adiabatic Piston



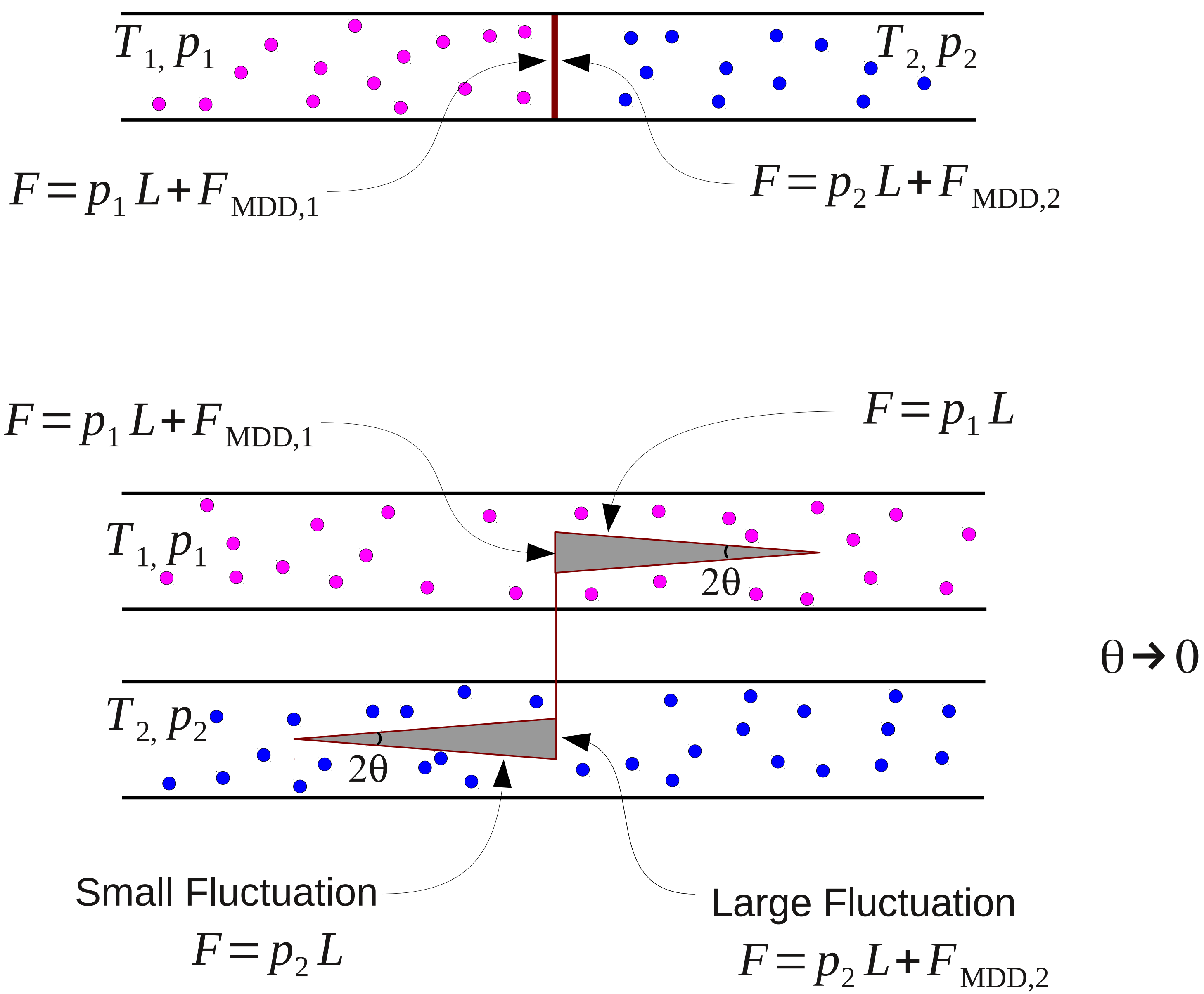
$$p_1 = p_2\,, \quad T_1 > T_2\,, \quad n_1 < n_2$$

$$F_{\mathrm{MDD},1} = -\sqrt{\frac{\pi}{8}}\frac{J_{\mathrm{diss}}}{v_{\mathrm{th},1}} \qquad F_{\mathrm{MDD},2} = \sqrt{\frac{\pi}{8}}\frac{(-J_{\mathrm{diss}})}{v_{\mathrm{th},2}}$$

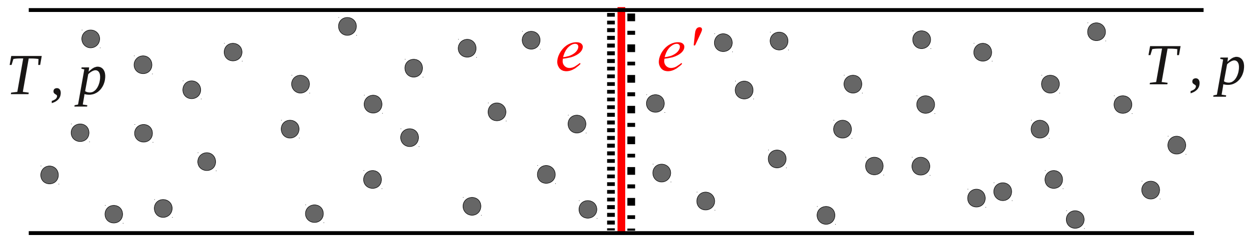
$$F_{\mathrm{NET}} = -\sqrt{\frac{\pi}{8}} J_{\mathrm{diss}} \left(\frac{1}{v_{\mathrm{th},2}} + \frac{1}{v_{\mathrm{th},1}} \right) = \sqrt{\frac{\pi}{8}} \frac{kT_1 - kT_2}{M\left(\gamma_1^{-1} + \gamma_2^{-1}\right)} \left(\frac{1}{v_{\mathrm{th},1}} + \frac{1}{v_{\mathrm{th},2}} \right)$$

- The piston moves in the opposite direction of heat.
- Momentum current from one gas to the other is compensated by the momentum the piston.

Adiabatic Piston vs Brownian Motor



Asymmetric Inelastic Piston



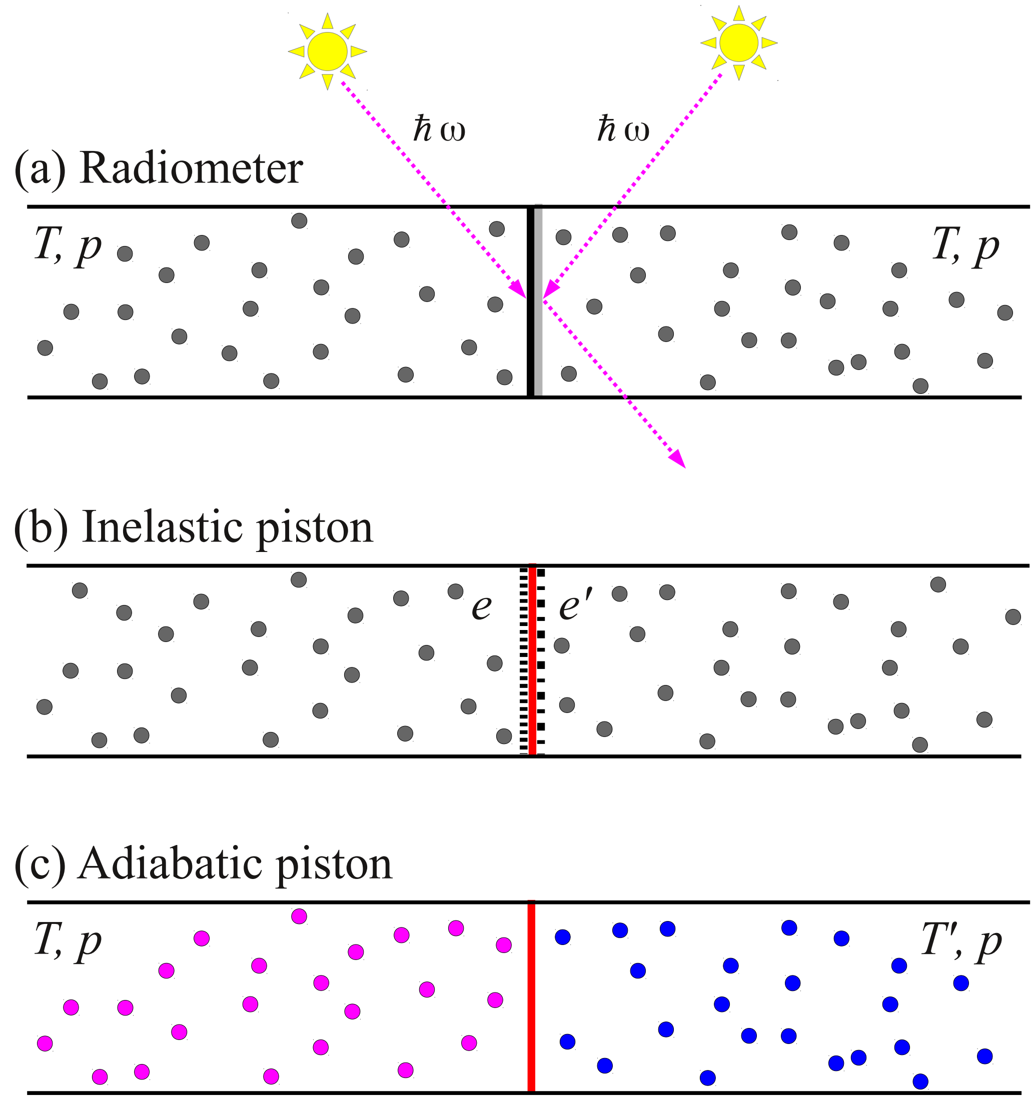
$$F_L = pL - \left(\frac{1}{2} + \frac{m}{M}\right)(1-e)\,pL \qquad F_R = -pL + \left(\frac{1}{2} + \frac{m}{M}\right)(1-e')\,pL$$

$$F_{\mathrm{NET}} = \left(\frac{1}{2} + \frac{m}{M}\right)(e-e')\,pL$$

The piston moves toward the smaller restitution coefficient.
(To the lossier side).

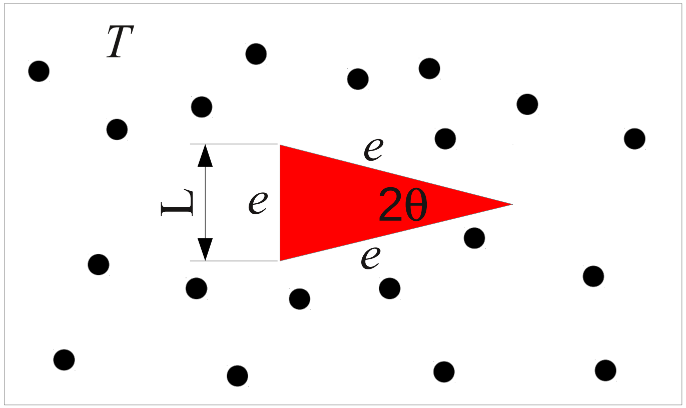
Fluctuation is not the main driving (similar to radiometer)

Costantini et al., EPL **82** (2008) 50008
Talbot et al., Phys. Rev. E **82** (2010), 011135



ℏω
ℏω
(a) Radiometer
T, p
T, p
(b) Inelastic piston
e
e′
(c) Adiabatic piston
T, p
T′, p

Granular Brownian Ratchet



$$F_{\mathrm{BASE}} = P\,L - \frac{1-e}{2} P\,L - \frac{m}{M}(1-e)\,p\,L$$

$$F_{\mathrm{SIDE}} \approx -P\,L + \frac{1-e}{2} P\,L + o(\theta)$$ (Fluctuation is negligible)

$$F_{\mathrm{NET}} = -\frac{m}{M}(1-e)\,p\,L < 0$$

Cleuren and Van den Broeck, EPL **77** (2007) 50003

Costantini et al., Phys. Rev. E **75** (2007), 061124

Conclusions

- Concept of Momentum Deficit due to Dissipation (MDD) Is introducded.
- Force by MDD $$F_{\mathrm{MDD}} = -c \frac{J_{\mathrm{diss}}}{v_{\mathrm{th}}}$$
- MDD captures the asymmetry in fluctuation
- Adiabatic piston, Brownian Motors, and Inelestic Brownian Ratchet are intuitively explained by MDD